

IV. Methodus quadrandi genera quædam Curvarum,  
aut ad Curvas Simpliciores reducendi. per A. De  
Moiſre R. S. S.

SIT *A* Area Curvæ cujus Abſciſſa  $x$ , & ordinatim Applica-  
ta  $x^m \sqrt{dx-xx}$ . Sit *B* Area Curvæ cujus Abſciſſa eadem  
cum priori, ſed ordinatim Applicata  $x^{m-n} \sqrt{dx-xx}$ ; ponatur  
 $\sqrt{dx-xx} = y$ . Erit Area *A* =

$$d^n B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} \text{ in } \frac{2m-3}{2m} \text{ in } \frac{2m-5}{2m-2} \&c. = P$$

$$- \frac{1}{m+2} x^{m-1} y^3 = -Q$$

$$- \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} x^{m-2} y^3 = -R$$

$$- \frac{dd}{m} \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} x^{m-3} y^3 = -S$$

$$- \frac{d^3}{m-1} \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} \text{ in } \frac{2m-3}{2m} x^{m-4} y^3 = -T$$

&c.

Ubi notandum 1° quod  $n$  Supponitur numerus integer &  
affirmativus ; 2° Quod Quantitas  $d^n B$  in ſerie per *P* deſig-  
nata, multiplicari debet in tot terminos quot ſunt unitates  
in  $n$  ; 3° quod tot ſequentes ſeries per  $-Q$ ,  $-R$ ,  $-S$ ,  $-T$   
&c. deſignatæ ſummi debeant, quot ſunt unitates in  $n$  ; quod ut

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Exem-

Exemplo uno vel altero clarius fiat, dico quod si  $n = 1$ , tunc  $A$

$$= d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3 \text{ \& si } n=2,$$

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-2}{2m+2} - \frac{1}{m+2} x^{m-1} y^3$$

$$- \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} \text{ in } x^{m-2} y^3$$

4<sup>o</sup> quod si  $y$  ponatur  $= \sqrt{dx-xx}$ , tunc  $A$  erit  $= Q - R + S - T \&c. + P.$

### Corollarium.

Si  $m$  ponatur æqualis termino cuivis sequentis Seriei

$$- \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2} \&c.$$

quadratura Curvæ cujus ordinatim Applicata  $x^m \sqrt{dx-xx}$ , aut  $x^m \sqrt{dx+xx}$  finita evadit & exhibetur per seriem nostram; quod ut Exemplo illustretur, Inquirenda sit Area Curvæ cujus ordinatim Applicata  $x^{-\frac{1}{2}} \sqrt{dx-xx}$ ; fingatur Curvam hanc comparari cum Curva cujus ordinatim Applicata  $x^{-\frac{3}{2}} \sqrt{dx-xx}$ , quoniam hoc in casu  $n=1$ , ideo

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3$$

sed  $m = -\frac{1}{2}$ , ergo  $2m+1=0$ , ideoq;

$$A = - \frac{1}{m+2} x^{m-1} y^3 = - \frac{2y^3}{3\sqrt{x^3}}$$

Hic Observatu dignum est quod Area sic reperta interdum data quantitate deficit a vera Area, aut eandem data quantitate excedit ; quo autem excessus iste aut defectus innotescat, supponatur Area reperta augeri minuive data quantitate  $q$ , tunc que posita  $x = 0$ , supponatur Area aucta minutave æqualis nihilo, sic in præsentī casu  $q$  reperietur  $= \frac{2}{3} d \sqrt{d}$ , adeoq;

$$A = \frac{2}{3} d \sqrt{d} - \frac{2 y^3}{3 \sqrt{x^3}}$$

### Corollarium 2<sup>dum</sup>.

Si  $n$  ponatur æqualis termino cuivis sequentis seriei 3, 4, 5, 6, 7, &c. Quadratura Curvæ cujus ordinatim applicata  $x^{-n} \sqrt{dx \cdot xx}$  aut  $x^{-n} \sqrt{dx + xx}$ , finita evadit, & exhibetur per seriem nostram ; Inquirenda fit Area Curvæ cujus ordinatim applicata  $x^{-3} \sqrt{dx \cdot xx}$ . finge eam comparari cum Area Circuli, quæ vocetur  $A$ ; erit  $m = 0$ ,  $n = 3$ , adeoq;  $A = P - Q - R - S$ . Sed cum quantitas  $2^m$  infinite parva seu potius nulla, in Denominatore termini tertii per quem  $d^n B$  multiplicatur, extet, Quantitas designata per  $P$  infinita est ; atque ob eandem causam, Quantitas designata per  $-S$  infinita evadit, adeoque Quantitates  $A$ ,  $-Q$ ,  $-R$  evanescunt : Igitur  $P = S$ ,

$$\text{divisæque æquatione per } \frac{2^m + 1}{2^m + 4} \text{ in } \frac{2^m - 1}{2^m + 2} \text{ fit}$$

$$d^n B \text{ in } \frac{2^m - 3}{2^m} = \frac{dd}{m} x^{m-3} y^3 \text{ seu } d^n B \text{ in } \frac{2^m - 3}{2}$$

$= dd x^{m-3} y^3$  : scriptisque 0 & 3 pro  $m$  &  $n$  prodibit

$$d B \text{ in } - \frac{3}{2} = \frac{y^3}{x^3}, \text{ seu } B = - \frac{2 y^3}{3 x^3},$$

### Corollarium 3<sup>um</sup>.

Si  $m$  ponatur æqualis termino cuivis sequentis seriei, — 2, — 1, 0, 1, 2, 3, 4, 5, &c. quadratura Curvæ cujus ordinata  $x^m \sqrt{dx-xx}$ , pendet a quadratura Circuli: Area vero Curvæ cujus ordinata  $x^m \sqrt{dx+xx}$  pendet a quadratura Hyperbolæ, & relatio istius Curvæ cum Circulo aut Hyperbola exhibetur per Seriem nostram in terminis finitis.

### Corollarium 4<sup>um</sup>.

Si  $m$  exponatur per alium quemvis terminum differentem ab iis quas supra memoravimus, Curva cujus ordinata  $x^m \sqrt{dx-xx}$  aut  $x^m \sqrt{dx+xx}$ , neque quadratur exacte, nec ab Hyperbola aut Circulo pendet, sed ad Curvam simpliciore[m] reducitur per seriem nostram.

### Theorema 2<sup>um</sup>.

Sit  $A$  Area Curvæ cujus Abscissa  $x$  & ordinatim applicata  $\frac{x^m}{\sqrt{dx-xx}}$ . Sit  $B$  area Curvæ cujus Abscissa eadem cum priori sed ordinatim applicata  $\frac{x^{m-n}}{\sqrt{dx-xx}}$  ponatur  $\sqrt{dx-xx} = y$ . Erit  $A =$

$d^n B$

$$d^n B \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} \text{ in } \frac{2m-7}{2m-6} \text{ } \phi.c. = P.$$

$$- \frac{1}{m} x^{m-1} \quad y = - Q.$$

$$- \frac{d}{m-1} \text{ in } \frac{2m-1}{2m} x^{m-2} \quad y = - R$$

$$- \frac{dd}{m-2} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} x^{m-3} \quad y = - S$$

$$- \frac{d^3}{m-3} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} x^{m-4} \quad y = - T$$

$\phi.c.$

Observationes ad primum Theorema, hic & in sequentibus locum habent.

### Corollarium I<sup>um</sup>.

Si  $m$  ponatur æqualis Termino cuivis sequentis seriei,

$$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \text{ } \phi.c. \text{ quadratura Curvæ cujus ordinatim ap-}$$

plicata  $\frac{x^m}{\sqrt{d \, x \cdot x \cdot x}}$  aut  $\frac{x^m}{\sqrt{d \, x + x \cdot x}}$  finita evadit, & exhibetur per hanc  
seriem.

Corollarium 2<sup>um</sup>.

Si  $n$  ponatur æqualis Termino cuivis sequentis seriei 1, 2, 3, 4, 5, 6, 7, &c. Curva omnis cujus ordinatim applicata

$$\frac{x^{-n}}{\sqrt{dx-xx}} \text{ aut } \frac{x^{-n}}{\sqrt{dx+xx}} \text{ quadratur per hanc seriem in terminis finitis.}$$

Corollarium 3<sup>um</sup>.

Si  $m$  exponatur per terminum quemlibet sequentis seriei, 0, 1, 2, 3, 4, 5, 6, 7, &c. Curva cujus ordinatim appli-

cata  $\frac{x^m}{\sqrt{dx-xx}}$  pendet a Quadratura Circuli. Curva vero cujus

ordinatim applicata  $\frac{x^m}{\sqrt{dx+xx}}$  a quadratura Hyperbolæ. Etenim

si Centro  $C$ , Diametro  $AB = d$  describatur Circulus  $AEB$ , ac fumatur  $AD = x$ ; erecto  $DE$  normaliter, jungo  $CE$ . Sector  $AEC$  per  $\frac{1}{2}dd$  divisus æqualis est Areæ Curvæ cujus Ordinata

$\frac{x^0}{\sqrt{dx-xx}}$  Eodem modo, si Centro  $C$ , Transverso axi  $AB = d$ ,

describatur æquilatera Hyperbola  $A E$ , fumatur  $AD = x$ , erigatur  $DE$  ad angulos rectos, jungatur  $CE$ ; sector  $A C E$  per  $\frac{1}{2}dd$  divisus æqualis est Areæ Curvæ cujus ordinata

$$\frac{x^0}{\sqrt{dx+xx}}$$

Corol-

Corollarium 4<sup>um</sup>.

Si  $m$  ponatur æqualis Termino cuivis, qui non in limitationes

præcedentes cadat, Curva cujus ordinata  $\frac{x^m}{\sqrt{r^2 + xx}}$

neque quadratur exacte,, nec a Circulo aut Hyperbola pendet, sed ad Curvam simpliciore[m] reducitur.

Theorema 3<sup>um</sup>.

Sit  $A$  Area Curvæ cujus Abscissa  $x$ , ordinatim applicata  $x^m \sqrt{r^2 + xx}$ ,  
sit  $B$  area Curvæ cujus Abscissa itidem  $x$ , ordinatim applicata  
 $x^{m-2} \sqrt{r^2 + xx}$ , ponatur  $\sqrt{r^2 + xx} = y$ . Erit.  $A =$

$$r^{2n} B \text{ in } \frac{m-1}{m+2} \text{ in } \frac{m-3}{m} \text{ in } \frac{m-5}{m-2} \text{ in } \frac{m-7}{m-4} \text{ \&c.} = P.$$

$$- \frac{1}{m+2} x^{m-1} y = - Q$$

$$- \frac{rr}{m} \text{ in } \frac{m-1}{m+2} x^{m-3} y = - R$$

$$- \frac{r^4}{m-2} \text{ in } \frac{m-2}{m+2} \text{ in } \frac{m-3}{m} x^{m-5} y = - S,$$

\&c.

Corollarium 1<sup>um</sup>.

Si  $m$  exponatur per terminum quemvis sequentis seriei 1, 3, 5, 7 9, \&c. Quadratura Curvæ cujus ordinata  $x^m \sqrt{r^2 + xx}$  aut  $x^m \sqrt{r^2 + xx}$  finita evadit, \& exhibetur per hoc Theorema.

Corol-

*Corollarium 2<sup>um</sup>.*

Si  $n$  exponatur per terminum quemvis sequentis seriei 2, 3, 4, 5, 6, &c. Curva cujus ordinata  $x^{-2n} \sqrt{rr-xx}$  aut  $x^{-2n} \sqrt{rr+xx}$ , quadratur exacte per hoc Theorema.

*Corollarium 3<sup>um</sup>.*

Si  $m$  exponatur per Terminum quemvis sequentis seriei —2, 0, 2, 4, 6, 8, &c. Quadratura Curvæ cujus ordinata  $x^m \sqrt{rr-xx}$ , pendet a Circulo. Quadratura vero Curvæ cujus ordinata  $x^m \sqrt{rr+xx}$ , pendet ab Hyperbola.

*Corollarium 4<sup>um</sup>.*

Si  $m$  exponatur per Terminum quemvis differentem ab illis quos supra memoravimus, Curva cujus ordinata  $x^m \sqrt{rr-xx}$ , aut  $x^m \sqrt{rr+xx}$ , neque exacte quadratur, nec a Circulo aut Hyperbola pendet, sed ad simpliciores Curvam reducitur.

*Theorema 4<sup>um</sup>.*

Sit  $A$  Area Curvæ cujus abscissa  $x$ , ordinatim applicata  $x^m$ ,  
 $\frac{x^m}{\sqrt{rr-xx}}$ , Sit  $B$  Area Curvæ cujus Abscissa idem  $x$ , Ordinatum ap-  
 $\frac{x^{m-2n}}{\sqrt{rr-xx}}$   
 plicata  $\frac{x^{m-2n}}{\sqrt{rr-xx}}$  Erit  $A =$



$$r^{2n} B \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} \text{ in } \frac{m-7}{m-6} \&c. = P.$$

$$- \frac{r^{m-1}}{m} x^m y = - Q$$

$$- \frac{r^2}{m-2} \text{ in } \frac{m-1}{m} x^{m-3} y = - R$$

$$- \frac{r^4}{m-4} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} x^{m-5} y = - S$$

$$- \frac{r^6}{m-6} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} x^{m-7} y = - T.$$

&amp;c.

*Corollarium 1<sup>um</sup>.*

Si  $m$  exponatur per terminum quemvis sequentis seriei 1, 3, 5, 7, 9, &c. Quadratura Curvæ cujus ordinata

$$\frac{x^m}{\sqrt{rr-xx}} \quad \text{aut} \quad \frac{x^m}{\sqrt{rr+xx}}, \text{ per hoc Theorema habetur in finitis}$$

Terminis

*Corollarium 2<sup>um</sup>.*

Si  $n$  exponatur per terminum quemlibet sequentis seriei 1, 2, 3, 4, 5, 6, &c. Curva cujus ordinatim applicata

$$\frac{x^{n-2n}}{\sqrt{rr-xx}} \quad \text{aut} \quad \frac{x^{n-2n}}{\sqrt{rr+xx}} \text{ exacte quadratur per hoc Theorema}$$

M m m m m m m

Co

Corollarium 3<sup>um</sup>.

Si  $m$  exponatur per terminum quemvis sequentis seriei 0, 2, 4, 6, 8, 10, &c. Quadratura Curvæ, cujus ordinatim appli-

cata  $\frac{x}{\sqrt{rr-xx}}$  pendet a quadratura Circuli. Etenim si Centro  $C$  ra-

dio  $CA = r$  describatur Circulus  $AEG$ , sumatur  $CD = x$ , erigatur  $DE$  normalis ad  $CD$ , Jungatur  $CE$ : Sector  $CAE$  per  $\frac{1}{2} rr$  divisus æqualis est Areæ Curvæ cujus ordinatim ap-

plicata  $\frac{x^0}{\sqrt{rr-xx}}$ . Eodem modo si Centro  $C$ , Transverso samiaxi

$CA = r$ , describatur æqualatera Hyperbola  $EAM$ , ducatur  $CF$  ad  $AC$  perpendicularis  $= x$ , ducatur  $FE$  axi parallela donec occurrat Hyperbolæ in  $E$ , jungatur  $CE$ : sector Hyperbolicus  $ACE$  per  $\frac{1}{2} rr$  divisus æqualis est Areæ Curvæ cujus ordi-

natim applicata  $\frac{x^9}{\sqrt{rr+xx}}$

Corollarium 4<sup>um</sup>.

Si  $m$  exponatur per terminum quemlibet a præcedentibus

differentem, Curva cujus ordinata  $\frac{x^m}{\sqrt{rr-xx}}$  aut  $\frac{x^m}{\sqrt{rr+xx}}$  neque quadratur exacte, nec a Circulo aut Hyperbola pendet, sed ad Curvam simplicioream reducitur,

*Theorema 5<sup>um</sup>.*

Sit *A* Area Curvæ cujus abscissa *x*, ordinatim applicata

$\frac{x^m}{d - x}$ ; sit *B* Area Curvæ cujus abscissa itidem *x*, ejusq; ordinatim

applicata  $\frac{x^{m-n}}{d - x}$  Erit Area

$$A = d^n B - \frac{x^m}{m} - \frac{d x^{m-1}}{m-1} - \frac{dd x^{m-2}}{m-2} \&c.$$

Sit ordinatim applicata  $\frac{x^m}{d + x}$ , tunc Area erit =

$$A = \frac{x^m}{m} - \frac{d x^{m-1}}{m-1} + \frac{dd x^{m-2}}{m-2} \&c. \pm d^n B.$$

*Corollarium.*

Si *m* exponatur per terminum quemlibet sequentis seriei, 0, 1, 2, 3, 4, 5, 6, &c. Quadratura Curvæ cujus ordinatim applicata  $\frac{x^m}{d - x}$ , aut  $\frac{x^m}{d + x}$  pendet a quadratura Hyperbolæ;

*Vide Fig. 3.*

Etenim ductis *D E*, *E F* ad angulos rectos, sumatur *E G* = *d*, ducatur *G H* normalis ad *E F* & ipsi æqualis. Intra Asymptotos *D E*, *E F* describatur Hyperbola per *H* transiens, quo facto sumatur *G K* = *x* versus *E* pro primo casu, at versus *F* pro secundo; ducatur ordinatim applicata *K L*: Area *H G K L* per *dd* divisa æqualis est Areæ Curvæ cujus ordinatim applica-

M m m m m m m 2

ta

$\frac{x^{\bullet}}{d-x}$  aut  $\frac{x^{\circ}}{d+x}$ . Hinc Solidum generatum a portione Cissoi-

dis dum circa Diametrum circuli genitoris revolvit, in finitis terminis exhibetur, data Hyperbolæ Quadratura.

### Theorema 6<sup>um</sup>.

Sit  $A$  Area Curvæ cujus abscissa  $x$ , ordinatim applicata

$\frac{x^m}{rr + xx}$ ; Sit  $B$  Area Curvæ cujus abscissa itidem  $x$ , ordinatim

applicata  $\frac{x^{m-2n}}{rr + xx}$ , Erit Area

$$A = \frac{x^{m-1}}{m-1} - \frac{rr x^{m-3}}{m-3} + \frac{r^4 x^{m-5}}{m-5} \&c. + r^{2n} B.$$

### Corollarium

Si  $m$  exponatur per terminum quemlibet sequentis seriei 0, 2, 4, 6, 8, &c. Quadratura Curvæ cujus ordinatim ap-

plicata  $\frac{x^m}{rr + xx}$  pendet a rectificatione circularis Arcus. Etenim si

centro  $C$  radio  $CA = r$  describatur Circulus  $AE G$ , ducatur Tangens  $AK = x$  jungatur  $C K$  peripheriæ occurrens in  $E$ ; arcus  $AE$  per  $rr$  divinus æqualis est Areæ curvæ cujus ordinata

$$\frac{x^{\circ}}{rr + xx}$$

Corol-

*Corollarium generale ad hæc sex Theoremata.*

Curva omnis mechanica cujus quadratura pendet ab aliqua  
Curvis numero infinitis, cujus ordinatæ formas sequentes adipisci

$$\text{possunt } x^m \sqrt{dx \pm xx}, - \frac{x^m}{\sqrt{dx \pm xx}}; x^m \sqrt{rr \pm xx} \frac{x^m}{\sqrt{rr \pm xx}}$$

$$\frac{x^m}{d \pm x}, \frac{x^m}{rr \pm xx}, \text{ per series has quadrari potest. Hoc Exemplo}$$

unico indicare satis erit.

Posito quod Cubus Arcus Circularis Sinui verso correspondentis fiat Ordinata Curvæ, cujus Abscissa sit idem Sinus versus. Inquirenda est Area istius Curvæ.

Sit Abscissa  $x$ , arcus circularis  $v$ , fluxio Areæ sit  $v^3 \dot{x}$ ,

Sit Area  $v^3 x - q$ . Igitur  $v^3 \dot{x} + 3 v^2 \dot{v} x - \dot{q} = v^3 \dot{x}$ ,

$$\text{unde } \dot{q} = 3 v^2 \dot{v} x; \text{ sed } \dot{v} = \frac{d\dot{x}}{2 \sqrt{dx - xx}}, \text{ igitur } \dot{q} = \frac{3 d v^2 x \dot{x}}{2 \sqrt{dx - xx}},$$

$$\text{sed per Theorema II. } \frac{x \dot{x}}{\sqrt{dx - xx}} = \frac{d\dot{x}}{2 \sqrt{dx - xx}} - \dot{v}; \text{ igitur } \dot{q} = \dot{v} - \dot{v},$$

$$\text{adeoque } \dot{q} = \frac{1}{2} d v^2 \dot{v} - \frac{1}{2} d v^2 \dot{v}; \text{ igitur } q = \frac{1}{2} d v^3 - \int \frac{1}{2} d v^2 \dot{v};$$

Ergo ad hoc perventum est ut fluentem quantitatem inveniamus cujus fluxio est  $\frac{1}{2} d v^2 \dot{v}$ .

Sit hæc quantitas  $\frac{1}{2} d v^2 y - r$ .

$$\text{Igitur } \frac{1}{2} d v^2 \dot{v} + 3 d v \dot{v} y - \dot{r} = \frac{1}{2} d v^2 \dot{v};$$

$$\text{Adeoque } \dot{r} = 3 d v \dot{v} y = \frac{3}{2} d d v \dot{x}; \text{ Sit } r = \frac{3}{2} d d v x - s.$$

$$\text{Igitur } \frac{3}{2} d d v \dot{x} = \frac{3}{2} d d v \dot{x} + \frac{3}{2} d d x \dot{v} - \dot{s};$$

$$\text{adeoque. } \dot{s} = \frac{3}{2} d d x \dot{v} = \frac{3 d^3 x \dot{x}}{4 \sqrt{dx - xx}} = \frac{1}{4} d^3 v - \frac{1}{4} d^3 \dot{v};$$

per 2<sup>um</sup> Theorema.

Igi-

Igitur  $s = \frac{1}{4} d^3 v - \frac{1}{4} d^3 y$ . adeoque area quæsitæ =  
 $v^3 x - \frac{1}{2} d v^3 + \frac{1}{2} d v^2 y - \frac{1}{2} d d v x + \frac{1}{4} d^3 v. - \frac{1}{4} d^3 y$ .

Quoniam autem Solida ex rotatione Curvarum genita, Superficies ab eadem rotatione genitæ, Longitudines Curvarum, & Centra Gravitatis horum omnium a Quadratura Curvarum pendent, hæc si a Curvis supradictis pendent facillime computantur.

Postquam Theoremata hæc concinnaveram, eaque Clarissimo *Newtono*, ut supremo harum rerum Judici, monstraveram; obtulit ille mihi Chartas suas manuscriptas, quibus mihi constat se diu compotem fuisse methodi qua, æquatione Trinomiali quavis data naturam Curvæ exprimente, illa Curva aut quadratur aut ad simpliciores Curvas reducitur.

Opandum autem esset ut non solum ea quæ ad hanc rem spectant, sed alia multa præclara ejus inventa publici juris facere dignaretur. Hoc credo univærsæ Reipublicæ Literariæ votum esse.

Nullus dubito Doctissimos viros quorum scripta in actis eruditorum alibique tam valde Mathematicas disciplinas promoverunt, methodos huic nostræ affines habere; adeoque nihil in his mihi ascribendum puto nisi quod Theoremata hæc reperierim, nescius an ullibi extarent; eaque ad formam tam facilem reducerim, ut calculus omnis ad hanc materiam spectans uno quasi intuitu conficiatur. Priusquam scribendi finem facio, non abs re futurum esse arbitror, si nunc, nulla data citius occasione, pauca quædam reposuerim Clarissimi *Leibnitii* animadversionibus, ad Seriem quandam a me publicatam de radice infinitæ æquationis inveniendæ. Existimat Vir Clar. Seriem illam non satis generalem esse, utpote non attingentem casus ubi quantitates  $z$  &  $y$  in se invicem ducuntur; adeoque seriem aliam pro mea substituit, hancq; asserit mea infinite generaliores: illum autem in levem hunc errorem inductum esse suspicor, quod quantitates  $a, b, c, d, \&c.$  pro quantitativibus datis assumpserit, cum pro quantitativibus datis aut indeterminatis indiscriminatim usurpandæ fuerint. Sed exemplum unum asserre libet, quo pateat seriem nostram casus omnes pervadere; sit Æquatio  $n y z - z^3 = y^3$  In Theoremate nostro fiat  $a = n y, b = 0, c = -1, g = 0, h = 0, i = 1$ , aut melius fiat  $g = y y$

$g = yy, h = 0, i = 0.$  in utroque casu fiet  $Z =$

$$Z = \frac{yy}{n} + \frac{y^3}{n^4} + \frac{3y^5}{n^7} + \frac{12y^{11}}{n^{10}} \&c.$$

## V.

*An Account of the Appearance of several Unusual  
Parhelia, or Mock-Suns, together with several  
Circular Arches lately seen in the Air by E. Halley.*

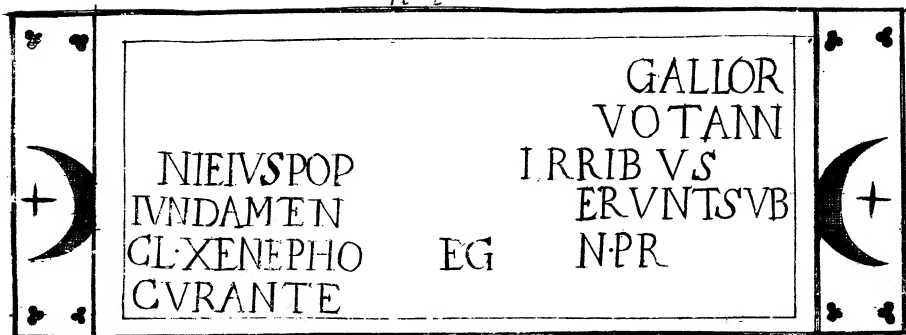
ON the Eighth of *April*, this present Year, 1702, walking in *London* Streets about ten in the Morning, the Air being clear, I observed the Sun to shine faintly, or as we call it, waterish; whereupon casting up my Eye, I perceived several Arches of Circles about him. I made what hast I could to get on the top of a House, which I did at Mr. *Mordens* by the Royal Exchange, and found the Appearance as is described in Figure 4. *Tab. 3<sup>o</sup>* wherein

*S* is the true Sun, *Z* the Zenith.

*S T P P* a great white Circle passing through the Sun, and as near as I could judge, parallel to the Horizon. It was very distinct and entire, about two Degrees broad in the *Northern* part about *T*; and held much the same breadth in the *East* and *West*, but grew narrower towards the Sun, its edges were not very well defined, the whole appearing like a faint white Cloud, and a part of it would have been taken for such, but the whole Circle seen in the pure Azure Sky was a very surprizing sight.

*V N X Y* a Halo, or rather *Iris*, that was likewise an intire Circle, having the Sun for its Center. I measured the Semidiameter of this to be much about 22 Degrees: the breath of this Arch which was well defined, was by estimate equal to the Suns Diameter, and it was coloured with the Colours of the *Iris*, but nothing near so *vivid* as in the common Rainbow. The *Reds* were next the Sun, and the *Blews* in the outward Limb. Within this Circle the Sky appeared somewhat obscure, especially near the Arch; and I take it, that the cause of that

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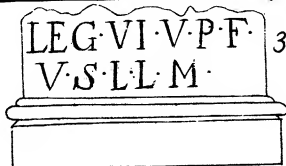
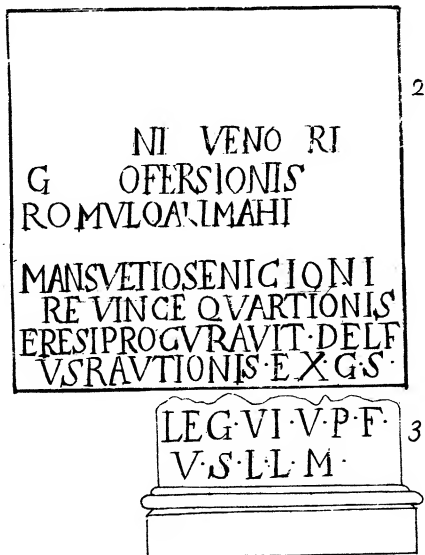
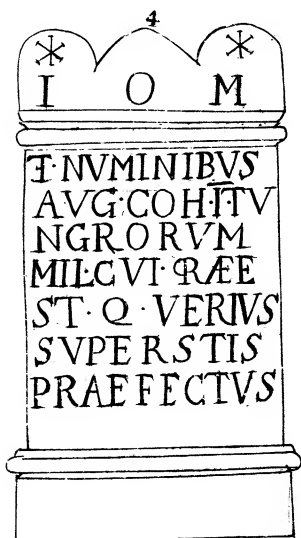
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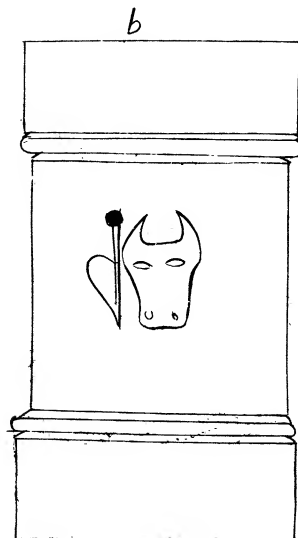
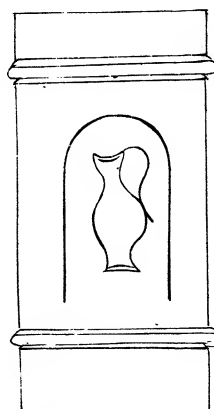
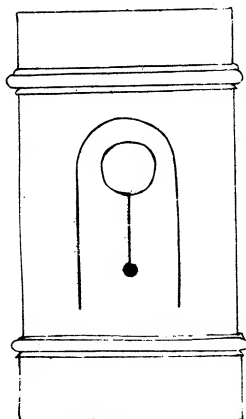
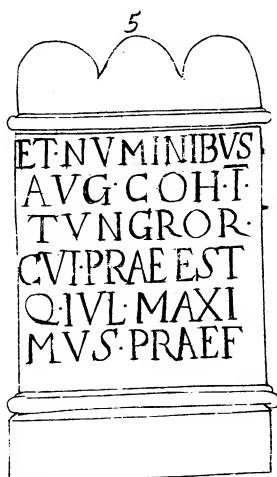
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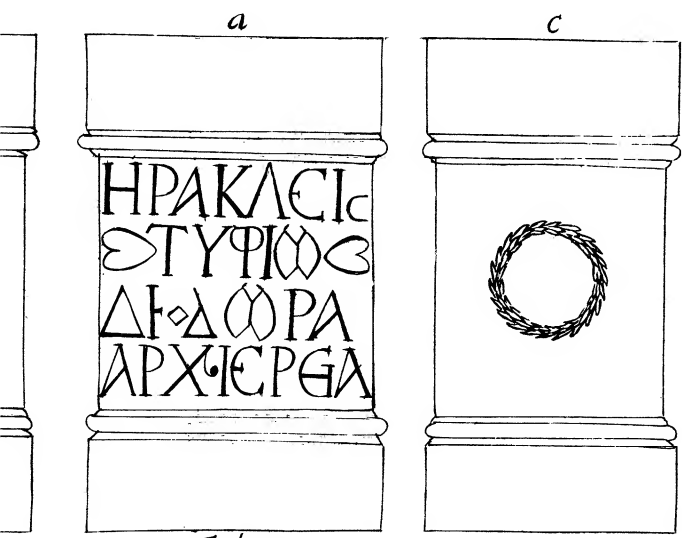
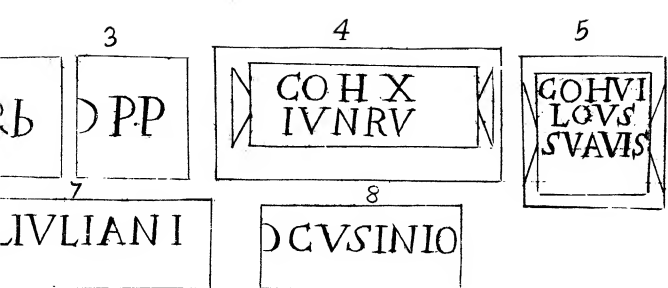


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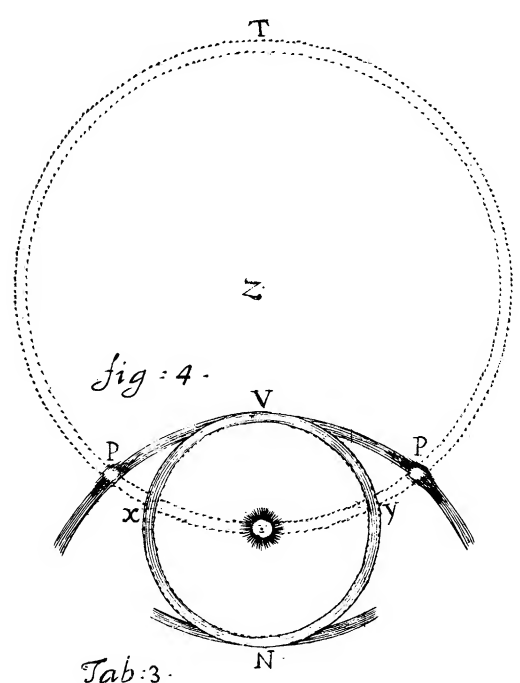
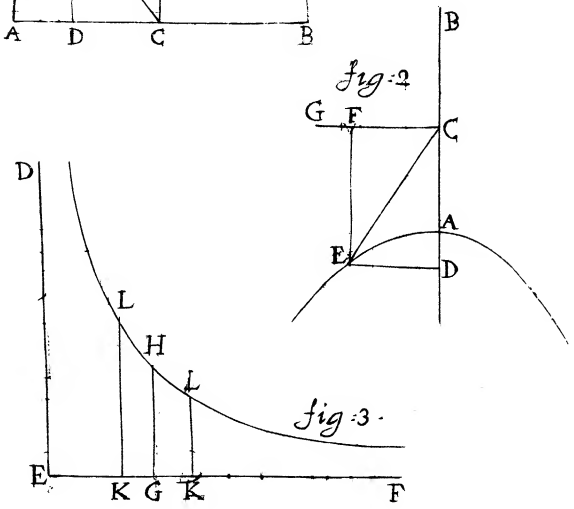
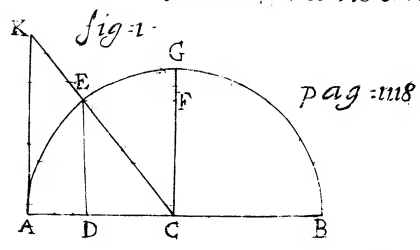


Tab. 1.





Tab:2.



Tab:3.

